Athena Transactions in Mathematical and Physical Sciences, Volume 1
Proceedings of the 1st International Symposium on Square Bamboos and the Geometree (ISSBG 2022), pp. 41–45
DOI: https://doi.org/10.55060/s.atmps.231115.004, ISSN (Online): 2949-9429
Proceedings home: https://www.athena-publishing.com/series/atmps/issbg-22

PROCEEDINGS ARTICLE

Geometric Figures Which Appear After VV Cutting in the Radial Cross Section of Generalized Möbius-Listing Bodies

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ABSTRACT

In previous works we were able to calculate all possible outcomes, and various options that appear, after VV, VS or SS cuts of GML bodies with the help of so-called straight chordal knives. Then we did not specify how many and what types of planar figures appear on the radial cross section of the GML_m^n body, depending on 1) m: the number of polygon vertices, 2) n: the number of twists, which also is a parameter showing which vertices (sides) are connected by this knife. In this article, a regularity is reported that allows to calculate the number and nature of planar figures appearing after an arbitrary VV in arbitrary regular m-gon. This work is another step towards solving the question whether it is possible to unequivocally restore the GML body knowing the information about the traces left on the radial cross section.

ARTICLE DATA

Article History
Received 20 August 2023
Revised 29 August 2023
Accepted 5 October 2023

Keywords
Analytic representation
Möbius-Listing bodies
Cutting of GML bodies

1. INTRODUCTION

Here we use all the traditional definitions and notation introduced in previous works [1,2,3]. In particular, VV cutting means cutting a polygon from vertex to vertex, with a chordal knife, which cuts the polygon in exactly two points. The following new parameters turn out to be decisive for these results.

1: VV_{1i} is the diagonal connecting the first and i-numbered vertices of the regular m-polygon. It is known from previous works that it suffices to consider $i = 2, \dots, [m/2]$, where $[m/2]$ is an integer part of a fraction.

2: $\kappa \equiv i - 1$ – number that is unique, for our needs, describing the given diagonal.

3: $j \equiv gcd(m, n)$ – this is a very important and informative parameter characterizing this GML_m^n body. If $j = 0$ we have the GML body without twisting or classical toroidal body.

We have identified three cases in which different patterns of different planar geometric figures in the radial cross section of the GML bodies are obtained.

Remarks:

In what follows, a polygon with *three* vertices is called a 3-gon, a polygon with 16 vertices is called a 16-gon etc. Furthermore, it should be noted that the methodology uses planar sections, whereas Generalized Möbius-Listing bodies are three dimensional. This means that the resulting ν-gons with the same color and shape form a single body after cutting.

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2. RESULTS

2.1. First Case

The diagonal contains the center of symmetry of the polygon (Fig. 1).

Then $m = 2k$ and $\kappa = k$, in this case for arbitrary $j = 1, \dots, m$ and the center of symmetry of the polygon is located on this diagonal.

Case 1A. If $j = m = 2k$ or $j = 0$, then two different, but identical plane $(k+1)$-gons appear after such VV cutting, but the Möbius phenomenon never occurs.

Case 1B. If $j = k$ and $m/j = 2$, then two identical plane $(k+1)$-gons appear after such VV cutting and the Möbius phenomenon always occurs.

Case 1C. If $j < k$, and m/j is even number, then m/j pieces identical $(j+2)$-gons appear after such VV cutting and the Möbius phenomenon always occurs.

Case 1D. If $j = 2\beta$ is an even number and $[m/j]$ is an odd number, then two different groups appear after VV cutting, each of which consists of $[m/j]$ pieces of $(\beta+2)$-gons!

2.2. Second Case

For arbitrary m when $\kappa \leq j$, then (m/j) similar $(\kappa+1)$-gons and one piece of $\left[m - (\kappa - 1)\frac{m}{j}\right]$-gon appears after VV cutting! (Fig. 2)

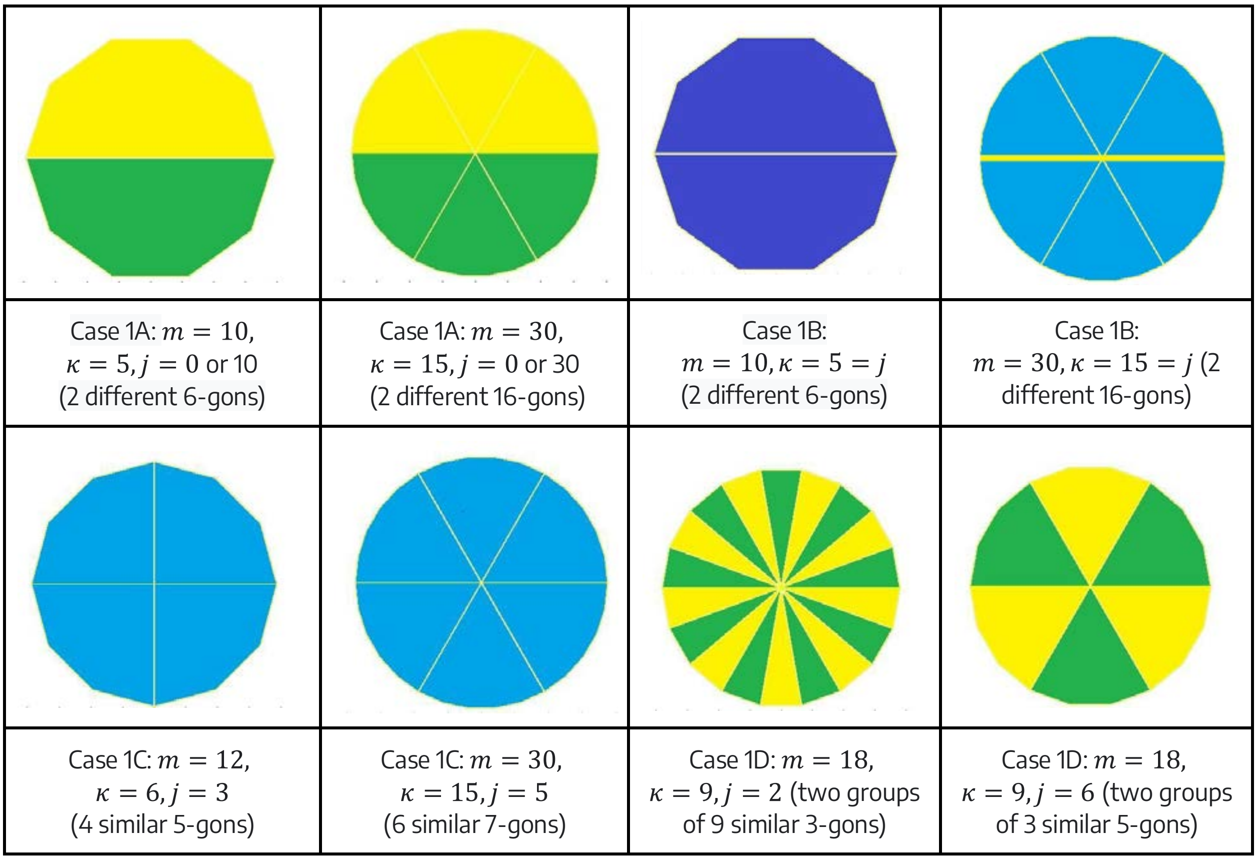


Figure 1. Examples for the first case with different parameter values.

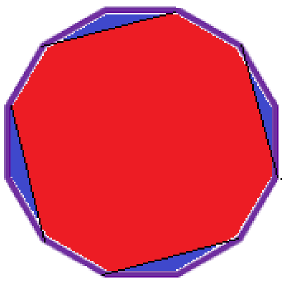
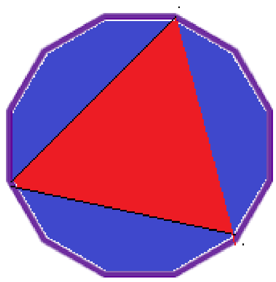
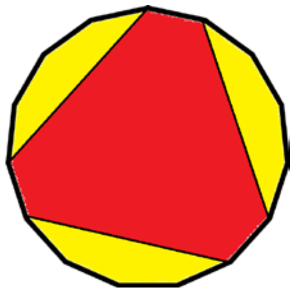
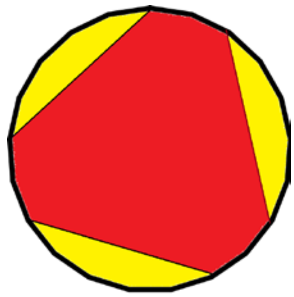
			
$m = 12, \kappa = 2 < j = 3$ 4 similar 3-gons and one 8-gon	$m = 12, \kappa = 4 \le j = 4$ 3 similar 5-gons and one 3-gon	$m = 15, \kappa = 4 \le j = 5$ 3 similar 5-gons and one 6-gon	$m = 21, \kappa = 5 \le j = 7$ 3 similar 6-gons and one 9-gon

Figure 2. Examples for the second case with different parameter values.

2.3. Third Case

For arbitrary m when $\kappa > j$. This turned out to be the most difficult case to study, which has many branches and shows a strong connection with the structure of numbers and geometric shapes.

Case 3.I. This subcase is considered separately, since for any values of m and n (even when these numbers are coprime) it is realized! For arbitrary m and $\kappa = 2, \ldots, [m/2]$ when $j \equiv 1$, then two different groups, each of which consists of $[m/j]$ pieces 3-gons and $(\kappa - 2)$ different groups, each of which consists of $[m/j]$ similar 4-gons and one $[m/j]$-gon appears after such VV cutting! (Fig. 3)

Case 3.GA. For arbitrary m and $\kappa = j\beta < [m/2]$, where $\beta \in Z$ an integer and $\beta > 1$, then one group consisting of $[m/j]$ pieces 3-gons, $(\beta - 2)$ different groups, each of which consists of $[m/j]$ pieces 4-gons, one group consisting of $[m/j]$ pieces $(j + 2)$-gons and one piece of $[m/j]$-gon appears after cutting! (Fig. 4)

Case 3.GB. For arbitrary m and $\kappa = j\beta + l < [m/2], \beta > 1$ and $\beta \in Z$ where $l = 1,2, \ldots, (\beta - 1)$, then one group consisting of $[m/j]$ pieces 3-gons, $(\beta - 2)$ different groups each of which consists of $[m/j]$ pieces 4-gons, one group consisting of $[m/j]$ pieces $[j - (l - 4)]$-gons, one group consisting of $[m/j]$ pieces $(l + 2)$-gons and one piece of $[m/j]$-gon appears after cutting! (Fig. 5)

Case 3.GC. For arbitrary m and $j < [m/2]$, when $\kappa = j\beta + l$ and $\beta = 1, l = 1,2, \ldots, (j - 1)$ then one group consisting of $[m/j]$ pieces $[j - (l - 3)]$-gons and one group consisting of $[m/j]$ pieces $(l + 2)$-gons and one piece of $[m/j]$-gon appears after cutting! (Fig. 6)

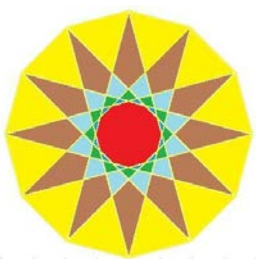
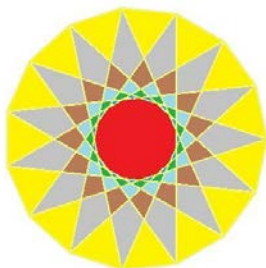
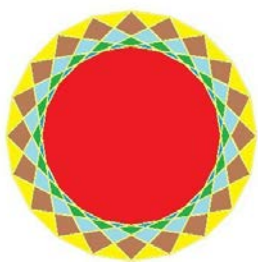
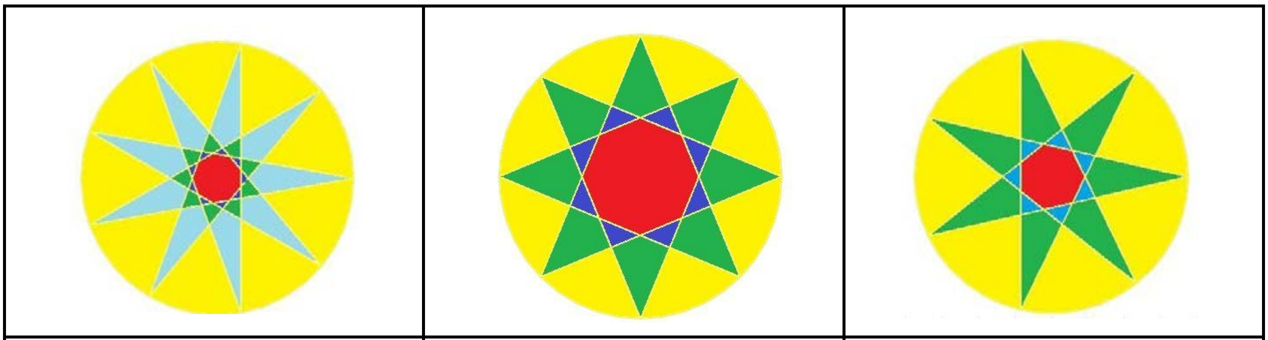
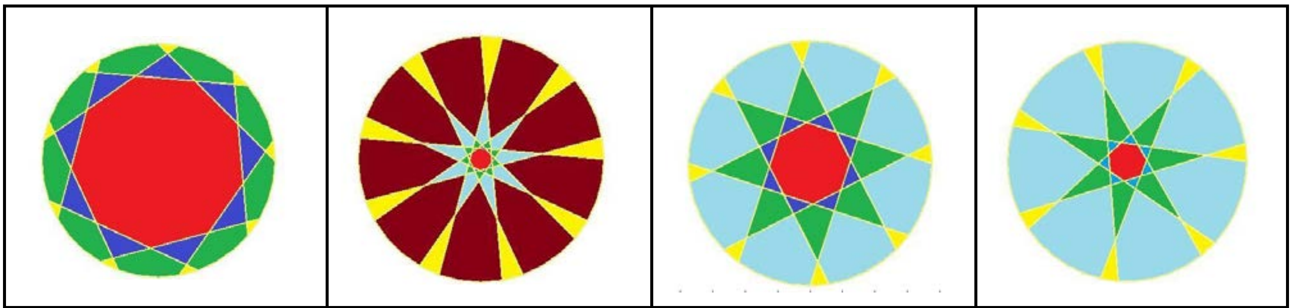
			
$m = 12, \kappa = 5 > j = 1$ 2-group (12 pieces of 3-gons); 3-group (12 similar 4-gons) and one 12-gon	$m = 15, \kappa = 6 > j = 1$ 2-group (15 pieces of 3-gons); 4-group (15 similar 4-gons) and one 15-gon	$m = 20, \kappa = 5 > j = 1$ 2-group (20 pieces of 3-gons); 3-group (20 similar 4-gons) and one 20-gon	$m = 20, \kappa = 8 > j = 1$ 2-group (20 pieces of 3-gons); 6-group (20 similar 4-gons) and one 20-gon

Figure 3. Examples for the third case with different parameter values, but j = 1.



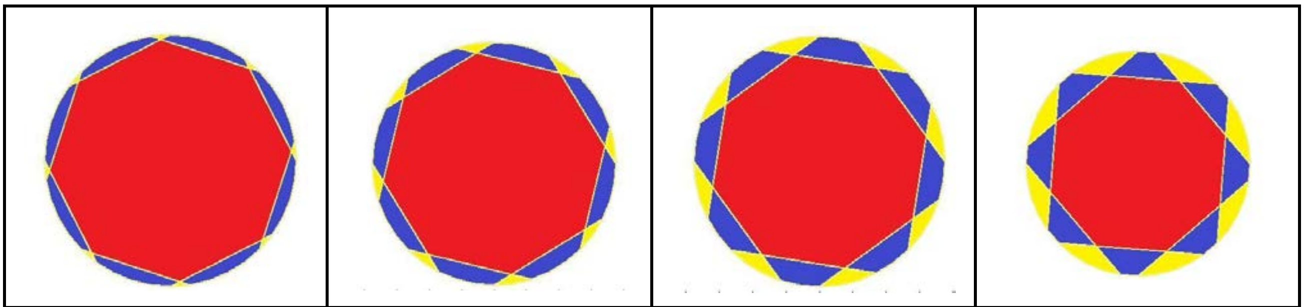
Case 3.GA: $m = 36, \kappa = 16 > j = 4$ 1-group (9 similar 3-gons), 2-group (9 similar 4-gons), 1-group (9 similar 6-gons) and one 8-gon	Case 3.GA: $m = 40, \kappa = 15 > j = 5$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gons), 1-group (8 similar 7-gons) and one 8-gon	Case 3.GA: $m = 42, \kappa = 18 > j = 6$ 1-group (7 similar 3-gons), 1-group (7 similar 4-gons), 1-group (7 similar 8-gons) and one 7-gon

Figure 4. Examples for the case 3.GA with different parameter values but always $\kappa = j\beta$.



Case 3.GB: $m = 36$, $\kappa = 9 > j = 4$ 1-group (9 similar 3-gons), 1-group (9 similar 7-gons), 1-group (9 similar 3-gons) and one 9-gon	Case 3.GB: $m = 36$, $\kappa = 17 > j = 4$ 1-group (9 similar 3-gons), 2-group (9 similar 4-gons), 1-group (9 similar 7-gons), 1-group (9 similar 3-gons) and one 9-gon	Case 3.GB: $m = 40$, $\kappa = 16 > j = 5$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gons), 1-group (8 similar 8-gons), 1-group (8 similar 3-gons) and one 8-gon	Case 3.GB: $m = 42$, $\kappa = 19 > j = 6$ 1-group (7 similar 3-gons), 1-group (7 similar 4-gon), 1-group (7 similar 9-gons), 1-group (7 similar 3-gons) and one 7-gon

Figure 5. Examples for the case 3.GB with different parameter values.



Case 3.GC: $m = 40$, $\kappa = 6 > j = 5$ 1-group (8 similar 7-gons), 1-group (8 similar 3-gons) and one 8-gon	Case 3.GC: $m = 40$, $\kappa = 7 > j = 5$ 1-group (8 similar 6-gons), 1-group (8 similar 4-gons) and one 8-gon	Case 3.GC: $m = 40$, $\kappa = 8 > j = 5$ 1-group (8 similar 5-gons), 1-group (8 similar 5-gons) and one 8-gon	Case 3.GC: $m = 40$, $\kappa = 9 > j = 5$ 1-group (8 similar 4-gons), 1-group (8 similar 5-gons) and one 8-gon

Figure 6. Examples for the case 3.GC with different parameter values.

3. FINAL REMARK

It should be obligatorily noted that at present this regularity has been discovered and tested on many examples of parameters, but by this time there is no complete mathematical proof. Therefore, we call this regularity a "*hypothetical regularity*". I also want to note that the situation is almost repeating itself, when in 2014 Johan Gielis and I found a general regularity about the number of GML_m^n-cutting bodies in different ways, and only in 2019 we were able to fully prove this! [1]

ACKNOWLEDGMENTS

The author expresses his deep gratitude to Johan Gielis for the constant and very attentive discussion of these issues and the constant belief that I will find a regularity, as well as for valuable advice. The author is very grateful to Levan Roinishvili, who is his former student, for creating a convenient and easy-to-use computer program, with the help of which examples of figures appearing after such cuts were built.

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