Chapter 7. Grandi (Rhodonea) Curves

The curves with polar equation $\rho = \cos(n\theta)$ $(0 \leq \theta \leq 2\pi)$ are also known as Grandi's roses, in honor of Guido Grandi who communicated his discovery to Gottfried Wilhelm Leibniz in 1713. Curves with polar equation $\rho = \sin(n\theta)$ $(0 \leq \theta \leq 2\pi)$ are equivalent to the preceding ones, up to a rotation of $\pi/(2n)$ radians.

As can be seen in Figure 10, Grandi's roses display n petals if n is odd and $2n$ petals if n is even. By using these polar equations it is impossible to obtain roses with $4n+2$ $(n \in \mathbf{N} \cup \{\mathbf{0}\})$ petals. Roses with $4n+2$ petals can be obtained by using the Bernoulli Lemniscate and its extensions. More precisely:

- The trigonometric function $y = \sqrt{\cos 2x}$ $(-\frac{\pi}{4} + k\pi \leq x \leq \frac{\pi}{4} + k\pi)$ becomes the so-called Bernoulli Lemniscate $\rho = \cos^{1/2}(2\theta)$ $(-\frac{\pi}{4} + k\pi \leq \theta \leq \frac{\pi}{4} + k\pi)$ $(k \in \mathbf{N})$ that is a rose with two petals (Figure 11).
- The functions $y = \sqrt{\cos(4n+2)x}$ $(n > 1)$ $(-\frac{\pi}{4(2n+1)} + \frac{k\pi}{2n+1} \leq x \leq \frac{\pi}{4(2n+1)} + \frac{k\pi}{2n+1})$ become the polar equations $\rho = \cos^{1/2}[(4n+2)\theta]$ $(-\frac{\pi}{4(2n+1)} + \frac{k\pi}{2n+1} \leq \theta \leq \frac{\pi}{4(2n+1)} + \frac{k\pi}{2n+1})$ $(k \in \mathbf{N})$ which give roses with $4n+2$ petals.

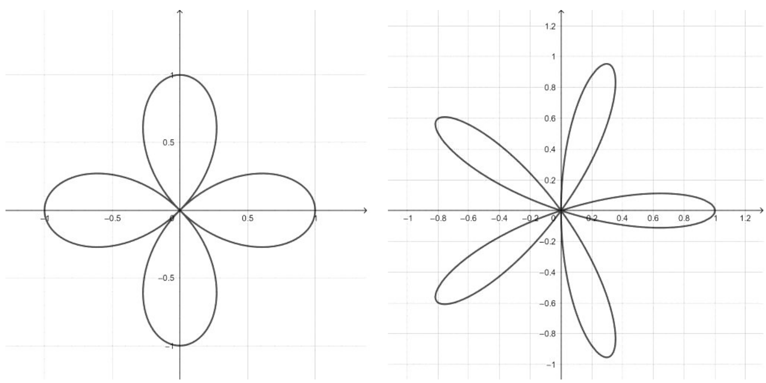

Figure 10. Rhodonea $\cos(2\theta)$ and $\cos(5\theta)$.

A few graphs of Rhodonea curves with fractional indices are shown in Figures 12–14.

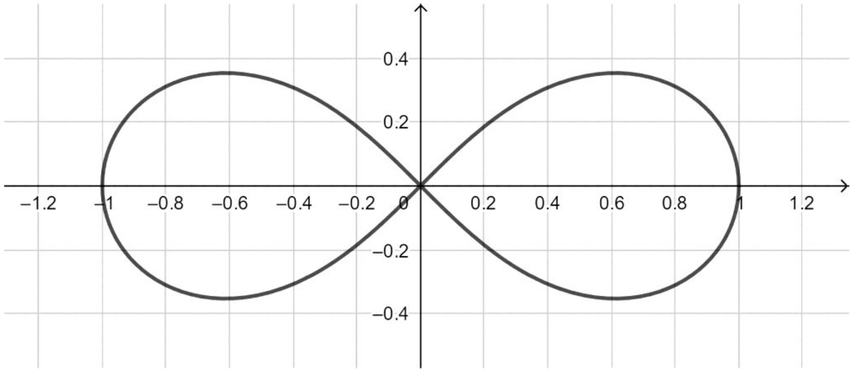

Figure 11. Bernoulli Lemniscate.

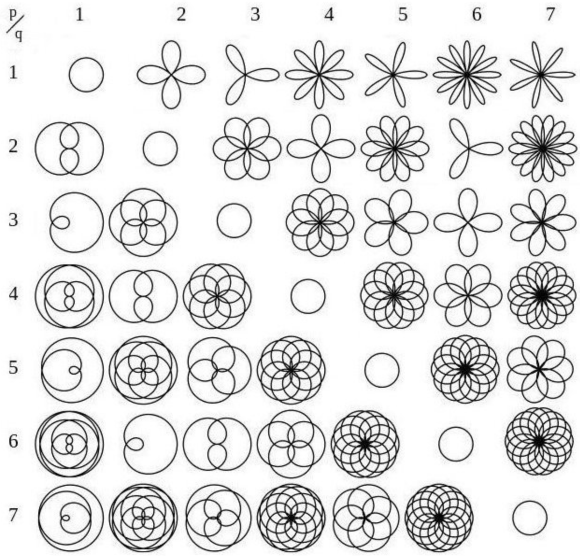

Figure 12. Rhodonea $\cos\left(\frac{p}{q}\,\theta\right)$.

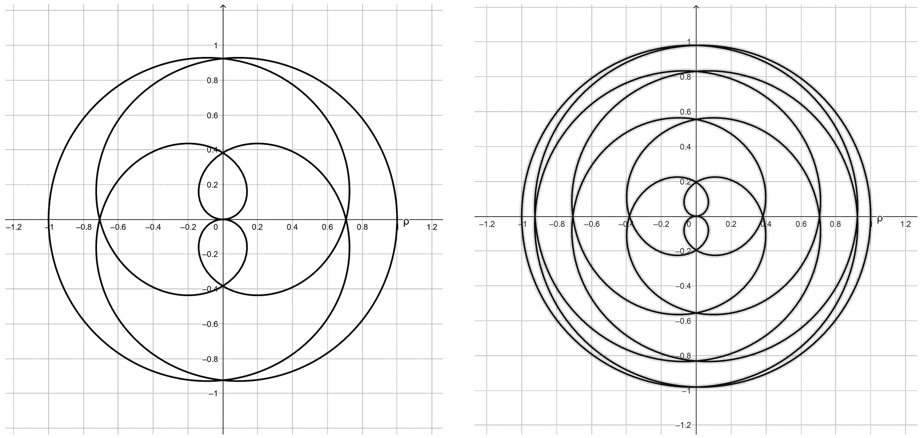

Figure 13. Rhodonea $\cos\left(\frac{1}{4}\,\theta\right)$ and $\cos\left(\frac{5}{4}\,\theta\right)$.

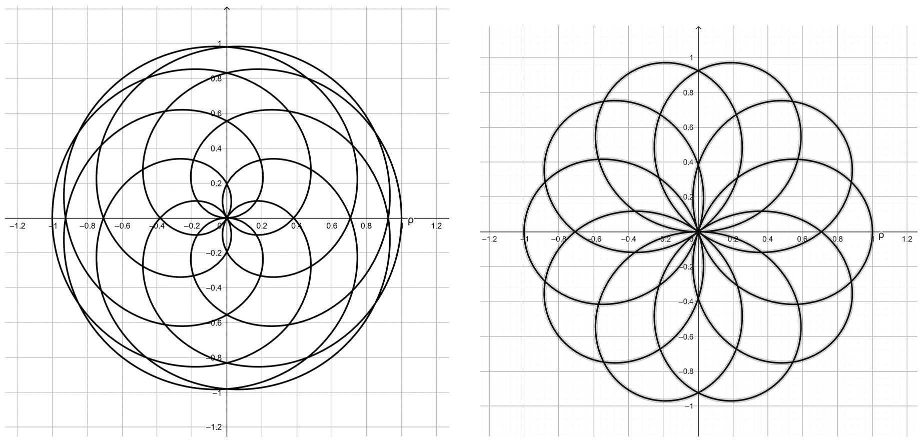

Figure 14. Rhodonea $\cos\left(\frac{1}{8}\,\theta\right)$ and $\cos\left(\frac{3}{8}\,\theta\right)$.